# The geometry of groups and rings and the $K$-theory of quotients of path algebras

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The topic Geometry and K-theory is addressed in 4 talks. The titles of the talks are found below. An overview of the talks is given next. Talk 1 constructs geometric objects which live inside groups and shows how to deform these objects one to another inside a group. The geometric objects include analogs of paths,1-loops, $n$-loops, $n$-tori, $n$-annuli, and $n$-manifolds in topology, $n \geq 1$. In a given (but arbitrary) group, the objects one recognizes as geometric objects depend on the circumstances in which the group is arising. Objects you recognize as paths, loops, etc in one circumstance may differ from objects you recognize as such in another circumstance, even though the abstract group is the same. This is explained in the talk. Once the geometric objects and their deformations are determined, you have by definition a *geometric group*. It is a very powerful tool which has wide impact. For example it enables one to present in terms of generators and relations every $K$-group $K_n(G)$, $n \geq 2$, of a classical-like group $G$ (for example a general linear group, a Chevalley group or a unitary group) providing $G$ is given the right geometty. This theorem follows from a main result of Talk 1, which presents every homotopy group $\pi_n(G)$, $n \geq 1$, of a geometric group $G$. The theorem for K-groups follows from this result because a $K$-group $K_n(G)$ of a classical-like group G is the homotopy group $\pi_{n-1}(C)$ of $G$ equipped with. the right geometry. Talks 2-4 develop the impact of these ideas on ring theory and on the interplay between group theory and ring theory. Talk 2 is an introduction to the geometry of rings. It defines the concept of a geometric ring and constructs its first homology group $H_1$. However, the main goal of Talk 2 is to begin a very fruitful interplay between the geometry of groups and the geometry of rings, leading to the concept of a generalized classical-like group. The interplay between geometric groups and geometric rings is carried forward in Talk 3. Talk 3 constructs the higher homology groups of a geometric ring and then a natural transformation from the homotopy group of a geometric group to the homology group of a corresponding geometric ring. Talk 4 constructs $K$-theory and homology groups for quotient algebras of a graph algebra (including of course Leavitt path and related algebras) without recourse to Talks 1-3 and then relates the results here to the geometry of groups and rings of Talks 1-3. Worth mentioning is that all of the talks employ directed graphs in various ways, including for the construction of geometric objects inside groups.

**Talk 1** The Geometry of Groups and Homotopy Theory

**Talk 2** Introduction to the Geometry of Rings and generalized Classical-like groups

**Talk 3** The Geometry of Rings and Homology Theory

**Talk 4** The K- and Homology Theory of Quotients of Path Algebras