The representation question and Steinberg algebras

Pere Ara
Universitat Autonoma de Barcelona

In this course, I will talk on a recent class of algebras which have arisen from the study of the representation problem for von Neumann regular rings, in recent work of Bosa, Pardo, Sims and the presenter. These algebras turn out to be closely connected to the Steinberg algebras of certain ample étale groupoids. For an ample étale groupoid $\mathcal{G}$, one may define a certain *type semigroup* $\mathrm{Typ}(\mathcal{G})$, which generalizes the type semigroup associated to an action of a discrete group on a totally disconnected topological space. This type semigroup is always a refinement monoid, i.e., it satisfies the Riesz refinement property. For each finitely generated conical refinement monoid M we build an ample Hausdorff étale topological groupoid $\mathcal{G}_M$ such that the type semigroup of $\mathcal{G}_M$ is canonically isomorphic to M. We can understand this groupoid as a categorification of M. The construction uses a classification result for such monoids in terms of combinatorial data, due to E. Pardo and the presenter, which allows us to represent them as monoids associated to certain separated graphs. We then are able to associate to each of these separated graphs a certain well-behaved inverse semigroup. Using this inverse semigroup, we follow standard techniques to build an ample étale Hausdorff groupoid $\mathcal{G}_M$ with the desired property that $\mathrm{Typ}(\mathcal{G}_M) \cong M$. Extending early constructions of Brustenga and the presenter, we aim to show that, for any field K, a suitable universal localization $\Sigma^{-1}A_K(\mathcal{G}_M)$ of the Steinberg algebra $A_K(\mathcal{G}_M)$ is von Neumann regular, and satisfies that its monoid of finitely generated projective modules is isomorphic to M. We also raise the question of whether the natural map $M \cong \mathrm{Typ}(\mathcal{G}_M) \to \mathcal{V}(C^*(\mathcal{G}_M))$ is injective.

In our talks, we will introduce all the concepts needed to understand this construction.